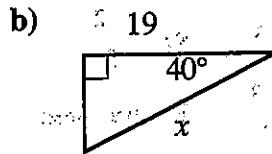
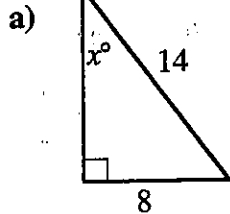


Trigonometry Lesson #5: The Sine Law

Review of Right Triangle Trigonometry

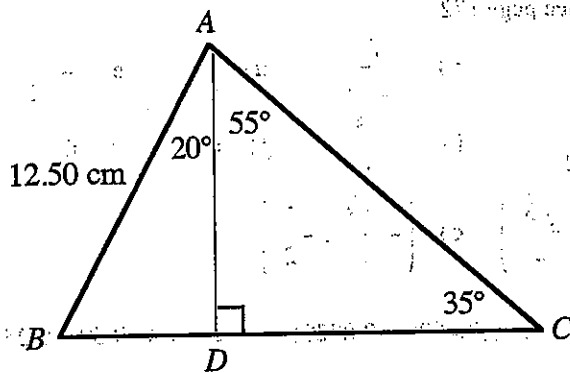
In previous courses, we studied trigonometry in right triangles using SOHCAHTOA.

In each case, determine the value of x to the nearest whole number.



Ex. #2

Use SOHCAHTOA to determine the length of BC . Work to three decimal places and answer to two decimal places.



Complete Assignment Questions #1 - #2

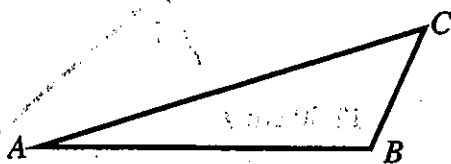
Trigonometry in Acute Angled and Obtuse Angled Triangles

In the next three lessons, we focus on solving triangles which are not right angled and in which SOHCAHTOA is not valid.

Often, in trigonometry, it is convenient to use the following notation.

In triangle ABC, represent

- the length of the side opposite angle A by a,
- the length of the side opposite angle B by b,
- and the length of the side opposite angle C by c.

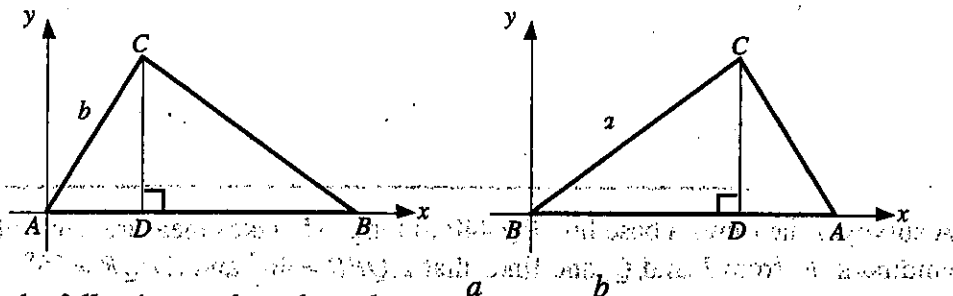


The Sine Law

In every triangle ABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ or $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Proof of the Sine Law

The diagrams show the same triangle ABC placed with base AB on the x-axis. In diagram i) the origin is at A, and in diagram ii) the origin is at B. The line CD is drawn perpendicular to AB.



Complete the following work to show that $\frac{a}{\sin A} = \frac{b}{\sin B}$

In i) $\sin A = \frac{CD}{AC} = \frac{CD}{b}$

In ii) $\sin B = \frac{CD}{BC} = \frac{CD}{a}$

$CD = b \sin A$

$CD = a \sin B$

It follows that $b \sin A = a \sin B$

Dividing both sides by $\sin A \sin B$ gives the result

Repeating the work above with AC placed on the x-axis would give the result $\frac{a}{\sin A} = \frac{c}{\sin C}$

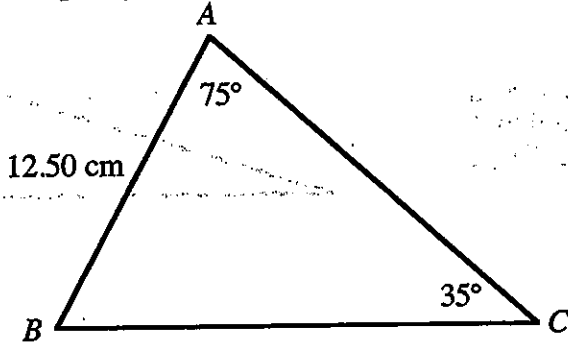
Hence $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ or $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Note

To use the sine law, we need to know **three** pieces of information. This information must include both numerator and denominator of one of the three fractions, i.e. we need to know an angle and the measure of its opposite side.

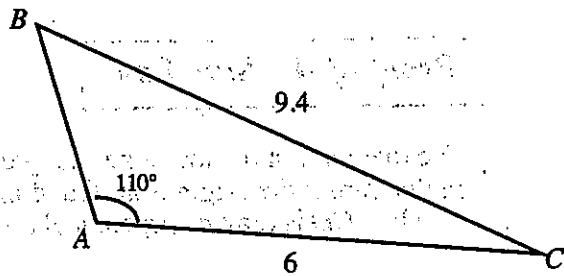
Ex. #3

Triangle ABC from Class Ex. #2 is shown. Use the sine law to calculate the length of BC , and compare your answer to the SOHCAHTOA method.



Ex. #4

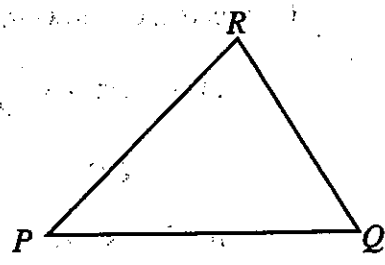
Use the sine law in the triangle shown to determine the measure of $\angle ACB$ to the nearest degree.



Ex. #5

A surveyor measures a base line PQ 440 m long. He takes measurements of a landmark R from P and Q , and finds that $\angle QPR = 46^\circ$ and $\angle PQR = 75^\circ$.

a) Calculate the perimeter of $\triangle PQR$ to the nearest metre.

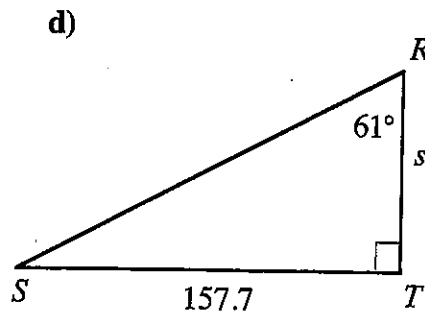
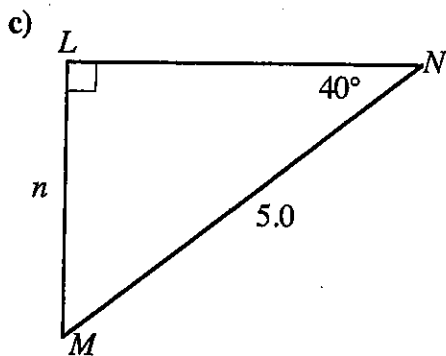
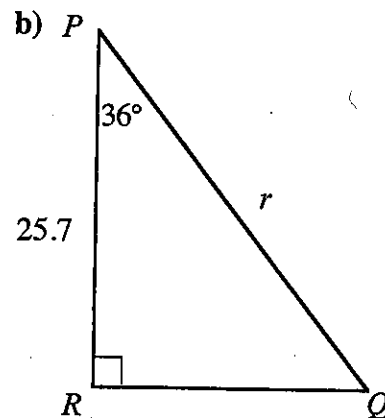
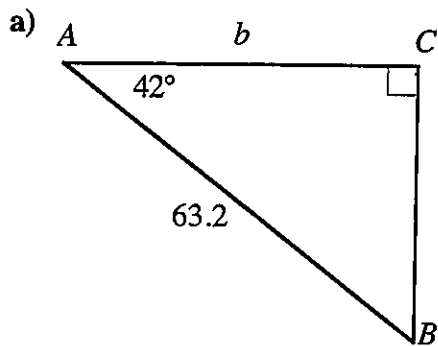


- b) Calculate the area of $\triangle PQR$ to the nearest hundred square metres.

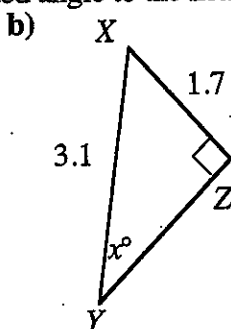
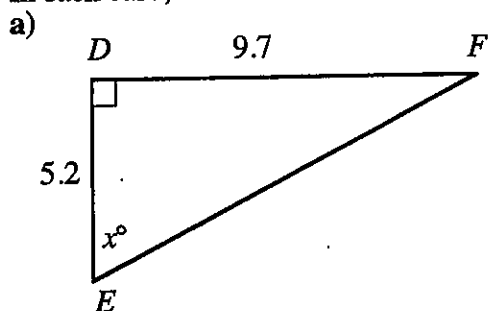
Complete Assignment Questions #3 - #11

Assignment

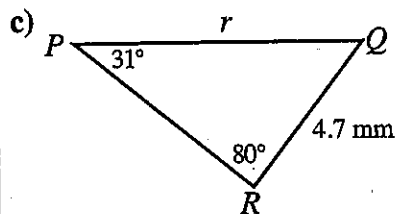
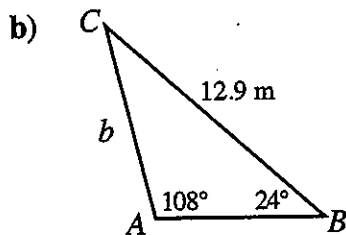
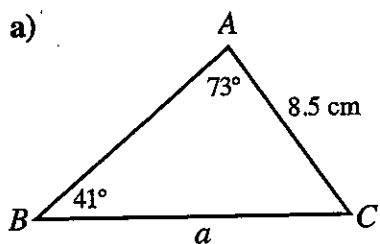
1. In each case, determine the length of the indicated side to the nearest tenth.



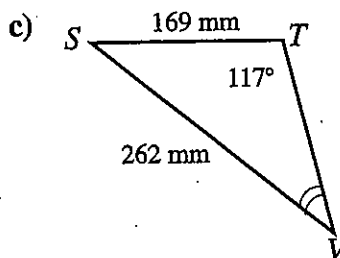
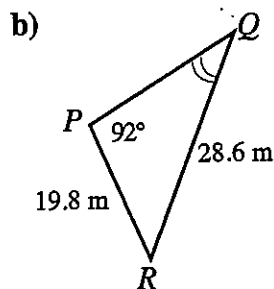
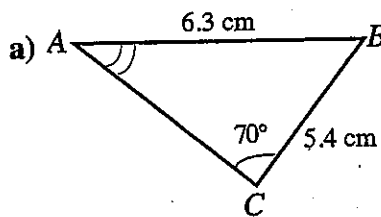
2. In each case, determine the measure of the indicated angle to the nearest degree.



3. In each case, find the length of the indicated side to the nearest tenth.

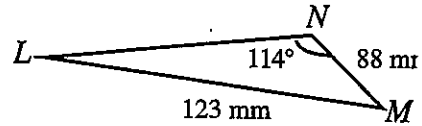


4. In each case, find the measure of the indicated angle to the nearest degree.



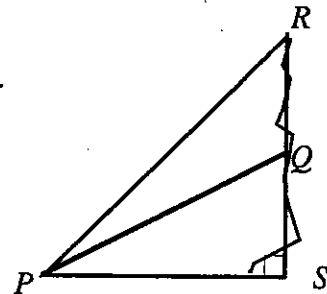
5. In $\triangle ABC$, angle $A = 49^\circ$, angle $B = 57^\circ$ and $a = 8$. Calculate b to the nearest tenth.

6. In $\triangle LMN$, angle $LMN = 114^\circ$, $LM = 123$ mm and $MN = 88$ mm. Calculate $\angle LMN$, to the nearest degree.

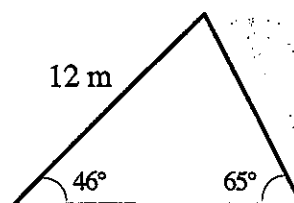


7. P and Q are two bases for a mountain climb. PQ is 600 m and QR is a vertical stretch of a rock face. The angle of elevation of Q from P is 31° , and the angle of elevation of R from P is 41° .

- a) Mark these measurements on the diagram and state the measure of angle PRQ .
- b) Use the sine law in $\triangle PQR$ to calculate the height of the vertical climb, QR , to the nearest metre.



8. Three students are trying to determine the area of the triangle in the diagram. Each student is given a different formula with which to determine the area. The area of the triangle is 53.3 m^2 .



Show how each student arrived at this answer.

Student #1: Use the formula $A = \frac{1}{2}bh$, where b is the length of the base and h is the vertical height.

Student #2: Use Heron's formula $A = \sqrt{s(s-a)(s-b)(s-c)}$, where a , b and c are the lengths of the three sides and s is the semi-perimeter of the triangle.

Student #3: Use the formula $A = \frac{1}{2}ab \sin C$, where a and b are the lengths of two sides and angle C is the contained angle between the sides a and b .

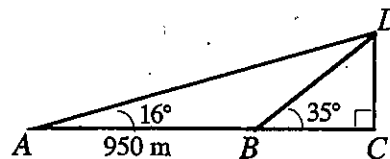
Multiple Choice

9. In triangle PQR , angle $P = 20^\circ$, angle $R = 150^\circ$ and $QR = 6$ m. The length of PQ is
- A. 4.1 m
 - B. 8.8 m
 - C. 15.2 m
 - D. 17.3 m

10. In $\triangle ABC$, $\angle A = 30^\circ$, $BC = 10$ units and $AC = 15$ units. If $\angle B$ is acute-angled, then $\angle C$ is
- A. 19.4°
 - B. 48.6°
 - C. 101.4°
 - D. 130.6°

Numerical Response

11. From a point A , level with the foot of a hill, the angle of elevation of the top of the hill is 16° . From a point B , 950 metres nearer the foot of the hill, the angle of elevation of the top is 35° . The height of the hill, DC , to the nearest metre, is _____.



(Record your answer in the numerical response box from left to right.)

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Answer Key

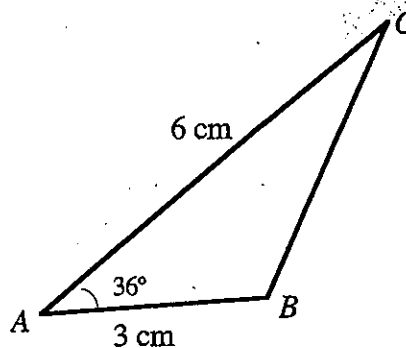
1. a) 47.0 b) 31.8 c) 3.2 d) 87.4 2. a) 62° b) 33°
 3. a) 12.4 cm b) 5.5 m c) 9.0 mm 4. a) 54° b) 44° c) 35°
 5. 8.9 6. 25° 7. a) 49° b) 138 m
 9. B 10. C 11.

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Trigonometry Lesson #6: The Cosine Law

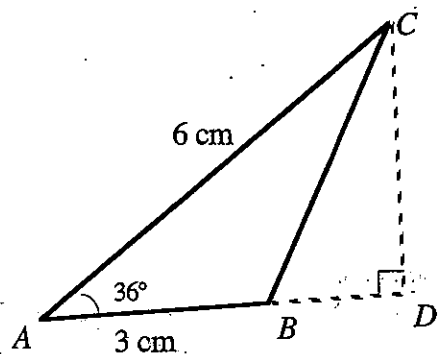
Warm-Up

Consider triangle ABC in which $\angle A = 36^\circ$, $AB = 3$ cm and $AC = 6$ cm. What happens when you try to apply the sine law to determine the length of BC ?



In the example above, where we are given the length of two sides and the contained angle, the sine law is **not** applicable.

Use SOHCAHTOA in the diagram below to determine the length of BC to the nearest tenth of a cm.



Note

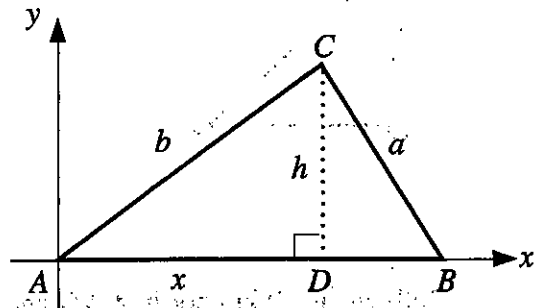
The method above is time consuming.
The length of BC can be determined in one step by using the **cosine law**.

The Cosine Law

In every triangle ABC , $a^2 = b^2 + c^2 - 2bc \cos A$.

Proof of the Cosine Law

- The diagram shows triangle ABC placed with base AB on the x -axis and A at the origin.
- The line CD is drawn perpendicular to AB and is h units in length.
- $AD = x$ units so $DB = c - x$ units.



Complete the following work to show that $a^2 = b^2 + c^2 - 2bc \cos A$.

In $\triangle ADC$ $\cos A = \frac{AD}{AC} = \frac{x}{b}$

In $\triangle BDC$ $BC^2 = CD^2 + DB^2$

$$a^2 = h^2 + (c - x)^2$$

$$a^2 = h^2 + c^2 - 2cx + x^2$$

$$a^2 = (h^2 + x^2) + c^2 - 2cx$$

$$a^2 = \quad + c^2 - 2c(\quad)$$

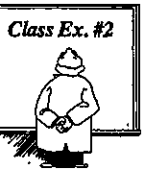
$$a^2 = b^2 + c^2 - 2bc \cos A$$

By placing AC and then BC on the x -axis, similar equations can be derived.

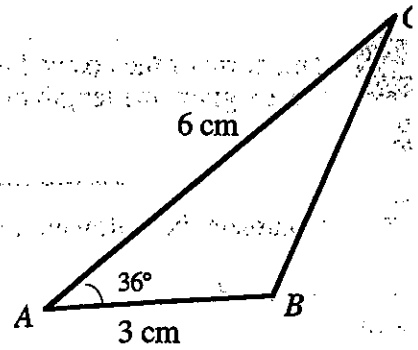
$$b^2 = c^2 + a^2 - 2ca \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

This version of the cosine law can be used in any triangle if we are given the length of two sides and the contained angle, (SAS).



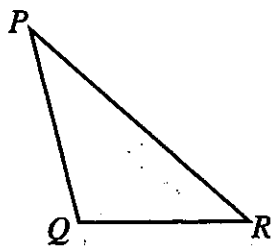
Consider the $\triangle ABC$ from Class Ex. #1 in which $\angle A = 36^\circ$, $AB = 3$ cm, and $AC = 6$ cm. Determine the length of BC , to the nearest tenth of a cm, using the cosine law.





Ex. #3

Find the length, to the nearest tenth of a cm, of the third side of $\triangle PQR$ if $QP = 1.7$ cm, $QR = 3.1$ cm and $\angle PQR = 110^\circ$.



Ex. #4

Bellevue is 30 km north of Ayr and Churchville is 18 km northwest of Ayr. Calculate the distance between Bellevue and Churchville to the nearest km.

Complete Assignment Questions #1 - #4

Alternative Form of the Cosine Law

The equation $a^2 = b^2 + c^2 - 2bc \cos A$

can be rearranged to the form $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$



Note

This form of the cosine law can be used to determine any angle in a triangle when we are given the length of all three sides, (SSS)



Ex. #5

Complete the following for triangle ABC .

- a) $\cos B =$
- b) $\cos C =$

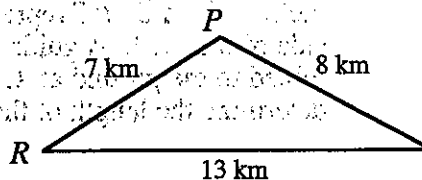
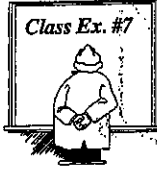
Class Ex. #6

Determine the largest angle in $\triangle ABC$ if $a = 14.7$, $b = 8.9$, and $c = 12.6$.



Class Ex. #7

Two ships set sail from port, P , heading in different directions. The first ship sails 7 km to R and the second ship sails 8 km to Q . If the distance between R and Q is 13 km, determine the angle between the directions of the two ships.



Complete Assignment Questions #5 - #12

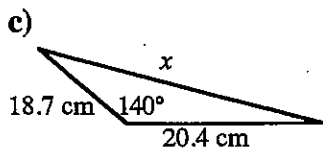
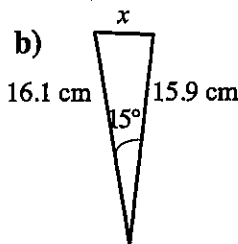
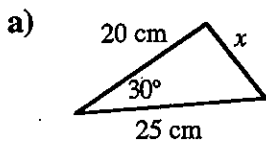
Assignment

1. Complete the following for triangle STV .

a) $s^2 =$

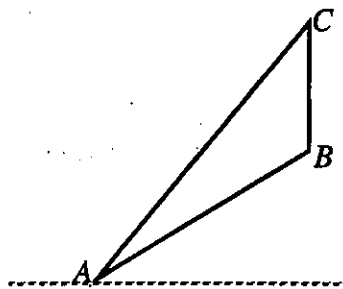
b) $v^2 =$

2. In each case, find the length of the indicated side to the nearest 0.1 cm.



3. In $\triangle ABC$, angle $A = 49^\circ$, $b = 24$ and $c = 37$. Calculate a to the nearest whole number.

4. In the diagram, AB represents part of a road constructed on the incline of a hill. BC represents a telephone pole 7.5 m tall at the side of the road. A guide wire attached to the top of the pole is joined to the ground at A . If $AB = 11.4$ m and $\angle ABC = 135^\circ$, determine the length of the guide wire to the nearest 0.1 m.



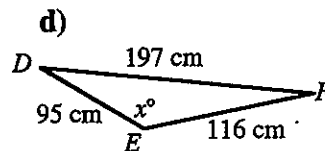
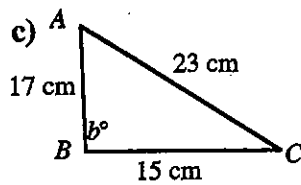
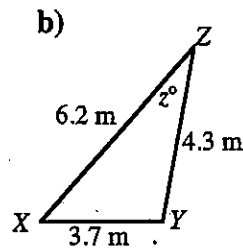
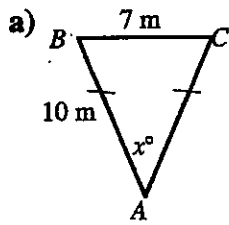
5. Solve triangle ABC in which $AB = 4.5$ cm, $BC = 7.8$ cm and angle $ABC = 79^\circ$. Round sides to the nearest tenth of a cm and angles to the nearest tenth of a degree.

6. Complete the following for triangle DEF .

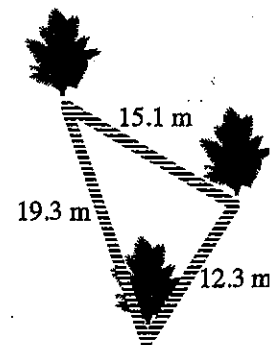
a) $\cos E =$

b) $\cos F =$

7. In each case, find the measure of the indicated angle to the nearest degree.

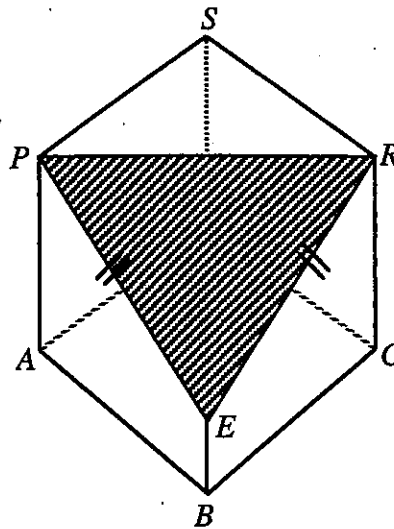


8. Anwar and Ingrid have three trees in their garden. The trees form a triangle as shown in the diagram. Determine the smallest angle between the trees.



9. The solid in the diagram was formed by removing a corner from a cube of 24 cm. The length of EB is 6 cm.

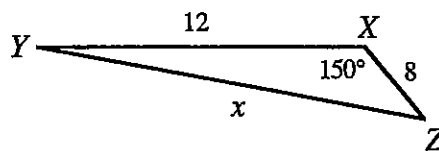
a) Calculate, to the nearest tenth, the lengths of PE and PR .



b) Calculate the measure of angle PER to the nearest degree.

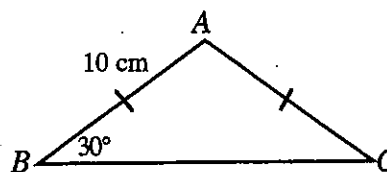
10. The value of x^2 is

- A. 112
- B. 304
- C. $208 - 96\sqrt{3}$
- D. $208 + 96\sqrt{3}$



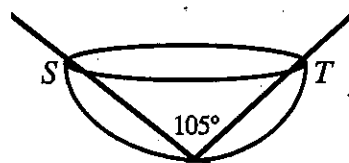
11. The length of BC in cm is

- A. $5\sqrt{3}$
- B. 10
- C. $10\sqrt{3}$
- D. 20



Numerical Response

12. The diagram shows a glass bowl with two chop-sticks resting on the rim at points S and T . The lengths of the parts of the chop-sticks inside the bowl are 9 cm and 11.5 cm respectively.



The length of ST , to the nearest tenth of a cm, is _____ .
 (Record your answer in the numerical response box from left to right.)

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Group Investigation

The sines of the angles of a triangle are in the ratio 2:3:4. Determine the ratios of the cosines of the angles.

Answer Key

1. a) $s^2 = t^2 + v^2 - 2tv \cos S$ b) $v^2 = s^2 + t^2 - 2st \cos V$
2. a) 12.6 cm b) 4.2 cm c) 36.7 cm 3. 28 4. 17.5
5. $\angle ABC = 79^\circ, \angle BAC = 68.5^\circ, \angle ACB = 32.5^\circ, AC = 8.2 \text{ cm}, BC = 7.8 \text{ cm}, AB = 4.5 \text{ cm}.$
 Answers may vary slightly depending on method.
6. a) $\cos E = \frac{d^2 + f^2 - e^2}{2df}$ b) $\cos F = \frac{d^2 + e^2 - f^2}{2de}$
7. a) 41° b) 36° c) 92° d) 138° 8. 40°
9. a) $PE = 30.0 \text{ cm}, PR = 33.9 \text{ cm}$ b) 69°
10. D 11. C 12.

1	6	.	3
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Group Investigation 14:11:-4